Soft Gluon Effects for W^+W^- and Higgs Vector Boson Associated Production at the LHC

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Fermilab

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Outline

Higgs Discovery

- Discovered a Higgs boson with mass $m_H \approx 125$ GeV.
- Era of directly probing the mechanism of EWSB has begun.
- Properties of Higgs are remarkably close to those of the SM Higgs.
- Must measure Higgs properties to highest accuracy.
- Many of the important Higgs production and decay channels are sensitive to additional heavy states and are directly related to EWSB.

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- Discovered a Higgs boson with mass $m_H \approx 125$ GeV.
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- Properties of Higgs are remarkably close to those of the SM Higgs.
- Must measure Higgs properties to highest accuracy.
- Many of the important Higgs production and decay channels are sensitive to additional heavy states and are directly related to EWSB.
- Beyond experimental precision, need precise theoretical predictions.
- $gg \to H$ known to NNLO in QCD and has been matched to threshold resummation.

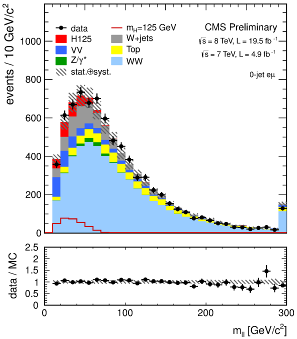
  Harlander, Kilgore, PRL88, 201801 (2002)
  Catani, de Florian, Grazzini, Nason, JHEP0307, 028 (2003)
  Anastasiou, Melnikov, NPB646, 220 (2002)
  Ravindran, Smith, van Neervan, NPB665, 325 (2003)
  Ahrens, Becher, Neubert, Yang, EPJ C62, 333 (2009)

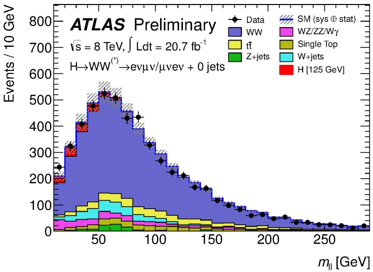
- Much progress in understanding jet vetoes theoretically

  Banfi, Salam, Zanderighi, JHEP1206, 159 (2012)
  Tackmann, Walsh, Zuberi, PRD86, 053011 (2012)
  Stewart, Tackmann, Walsh, Zuberi, 1307.1808
  Becher, Neubert, JHEP1207, 108 (2012)
  Liu, Petriello, PRD87, 014018 (2012); PRD87, 094027 (2013)

- Need to understand backgrounds also.

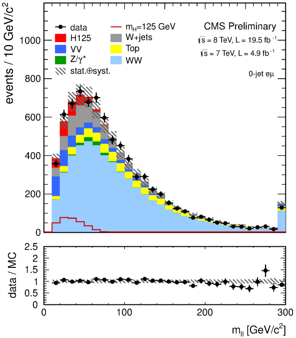
$H \to W^+W^-$

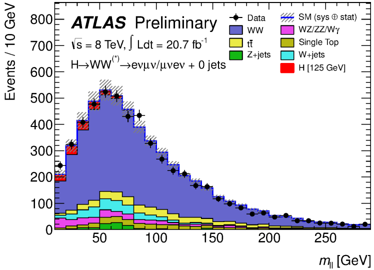




- Directly probes EWSB.
- Standard model $q\bar{q} \to W^+W^-$ is one of the major irreducible backgrounds for fully leptonic decay.
- Background analyzed via side-band analysis.

$H \to W^+W^-$

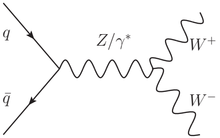


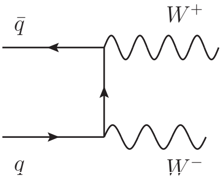


- Background analyzed via side-band analysis:
  - $H \to W^+W^-$ prefers lower di-lepton invariant mass, $m_{\ell\ell}$.
  - Analyze background rate at higher $m_{\ell\ell}$ in control region.
  - Use this to normalize Monte Carlos and extrapolate to lower $m_{\ell\ell}$.
- For this procedure to work, need to know differential distributions as well as possible.
- In particular, make sure $d\sigma/dM_{WW}$ is stable against higher order corrections.

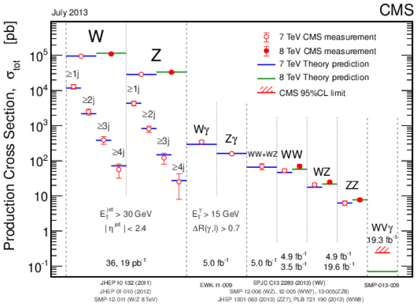
Trilinear Gauge Couplings

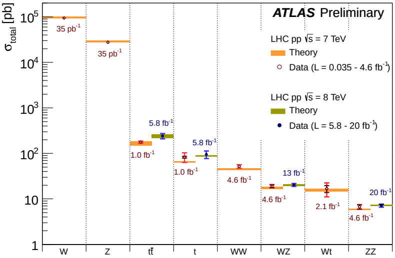
- $q\bar{q} \to W^+W^-$ interesting in its own right.
- Trilinear gauge couplings:
  - Measurement directly probes non-Abelian structure of EW sector.
  - To be sensitive to deviations in trilinear couplings, again, need accurate predictions for distributions.
- $pp \to W^+W^- \to \ell_1^+\ell_2^- + 2\nu$ hide new physics.
  - Large missing energy.
  - Anomaly...





W^+W^- Anomaly?





- 7 TeV:

$52.4 \pm 2.0\,(\text{stat.}) \pm 4.5\,(\text{syst.}) \pm 1.2\,(\text{lumi})$ pb

- 8 TeV:

$69.9 \pm 2.8\,(\text{stat.}) \pm 5.6\,(\text{syst.}) \pm 3.1\,(\text{lumi})$ pb

- 7 TeV:

$51.9 \pm 2.0\,(\text{stat.}) \pm 3.9\,(\text{syst.}) \pm 2.0\,(\text{lumi})$ pb

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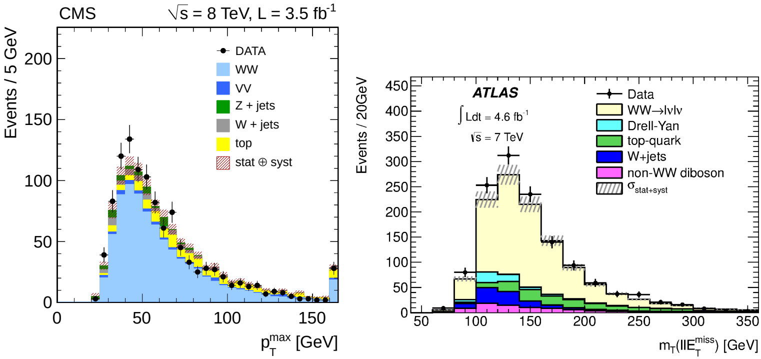
- SM from MCFM at NLO: 7 TeV: 47 ± 2 pb 8 TeV: $57.3^{+2.4}_{-1.6}$ pb
  Campbell, Ellis, Williams, JHEP1107, 190 (2013)
- W^+W^- consistently larger than SM prediction by $\sim 1-2\sigma$.
- Recent interest in new physics processes that could explain excess.
  Curtin, Jaiswal, Meade, 1206.6888; Rolbiecki, Sakurai, 1303.5696; Feigl, Rzehak, Zeppenfeld, 1205.3468

W^+W^- Anomaly?



- Is the higher order calculation under control?

Current Status

- Electroweak corrections known to NLO Bierweiler, Kasprzik, Kuhn, Uccirati, JHEP1211, 093 (2012) Baglio, Ninh, Weber, 1307.4331
  - Contribute less than $1-2\%$ to total cross section at LHC.
- $pp \to W^+W^-$ known at NLO in QCD Ohnemus, PRD44, 1403 (1991); Frixione, NPB410, 280 (1993) Dixon, Kunszt, Signer, NPB531, 3 (1998)
  - Increase LO cross section by $\sim 50\%$
  - $gg \to W^+W^-$ contributes another $\sim 3\%$ at 7 TeV and $\sim 4\%$ at 14 TeV. Dicus, Kao, Repko, PRD36, 1570 (1987); Glover, van der Bij, Phys. Lett. B219, 488 (1989) Binoth et. al. JHEP0612, 046 (2006); JHEP0503, 065 (2005)
  - $3-4\%$ uncertainty from pdfs and scale variation Campbell, Ellis, Williams, JHEP1107, 190 (2013)
- NLO QCD corrections incorporated in MCFM Campbell, Ellis, Williams, JHEP1107, 190 (2013)
- Interface with parton shower in POWHEG Melia, Nason, Rontsch, Zanderighi, JHEP1111, 078 (2011) Hamilton, JHEP1101, 009 (2011) Hoche, Krauss, Schonherr, Siegert, JHEP1104, 024 (2011)
- Will try to improve.

Threshold Logs

- In fixed order calculation, infrared finite results occur due to cancellation of real and virtual soft divergences.
- However, at edges of phase space large logs associated with these divergences appear.
  - Appear at every order in perturbation theory, and spoil perturbative convergence.
- For pair invariant mass distributions, interested in the partonic threshold.
  - $z = M^2/\hat{s} \sim 1$: $\alpha_s^n \dfrac{\ln^{2n-1}(1-z)}{1-z}$
  - M is invariant mass of hard process.
  - $\sqrt{\hat{s}}$ energy of total partonic process.
- Typically, these pieces give large contribution to higher order calculations.

Threshold Resummation

- QCD factorization allows us to factorize the collinear and hard physics:

$$\frac{d\sigma}{dMd\cos\theta} = \int_\tau^1 \frac{dz}{z} C(z, M, \cos\theta, \mu_f) \mathcal{L}\left(\frac{\tau}{z}, \mu_f\right),$$

  - Hard scattering kernel C
  - Parton luminosity $\mathcal{L}$
  - $z = M^2/\hat{s}$, $\tau = M^2/s$.
- Near partonic threshold have a new scale, the energy of soft emissions $\sqrt{\hat{s}}(1-z)$.

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  - Hard scattering kernel C
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  - $z = M^2/\hat{s}$, $\tau = M^2/s$.

- Near partonic threshold have a new scale, the energy of soft emissions $\sqrt{\hat{s}}(1-z)$.
- Have additional factorization between soft and hard scales near threshold:

$$C(z, M, \cos\theta, \mu_f) = \mathcal{H}(M, \cos\theta, \mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z), \cos\theta, \mu_f) + \mathcal{O}(1-z)$$

- Two portions:
  - Hard function $\mathcal{H}$: depends on scale of hard process M
  - Soft function $\mathcal{S}$: depends on energy of soft emitted gluons $\sqrt{\hat{s}}(1-z)$
- Separation of scales suggests and EFT approach.

SCET

- Near threshold:
$$\frac{d\sigma}{dMd\cos\theta} = \int_\tau^1 \frac{dz}{z}\mathcal{H}(M,\cos\theta,\mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z),\cos\theta,\mu_f)\mathcal{L}\left(\frac{\tau}{z},\mu_f\right),$$
- The appropriate EFT is Soft Collinear Effective Theory (SCET)
  Bauer, Fleming, Luke, PRD63, 014006 (2000)
  Bauer, Fleming, Pirjol, Stewart, PRD63, 114020 (2001)
  Bauer, Pirjol, Stewart, PRD65, 054022 (2002)
  Beneke, Chapovsky, Diehl, Feldmann, NPB643, 431 (2002)
  - Identify fields with soft and collinear momentum :
  $$k_c \sim Q(\lambda^2,1,\lambda), \quad k_{\bar{c}} \sim Q(1,\lambda^2,\lambda), \quad k_s \sim Q(\lambda^2,\lambda^2,\lambda^2)$$
  in basis $(n\cdot p, \bar{n}\cdot p, p_\perp)$
  - Hard QCD modes are “integrated out” of SCET by matching onto full QCD.
- Each component evaluated at their relevant scales:
  - Hard function is a Wilson coefficient evaluated at a hard scale μ_h
  - Soft function evaluated at a soft scale μ_s

SCET

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$$\frac{d\sigma}{dMd\cos\theta} = \int_\tau^1 \frac{dz}{z} \mathcal{H}(M,\cos\theta,\mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z),\cos\theta,\mu_f)\mathcal{L}\left(\frac{\tau}{z},\mu_f\right),$$

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  - Hard QCD modes are “integrated out” of SCET by matching onto full QCD.
- Each component evaluated at their relevant scales:
  - Hard function is a Wilson coefficient evaluated at a hard scale μ_h
  - Soft function evaluated at a soft scale μ_s
- Run components to common scale μ_f via renormalization group equations.
- By choosing $\mu_s \sim \sqrt{\hat{s}}(1-z)$, this running resums large logs.

SCET Resummation

- Using SCET, possible to perform resummation directly in momentum space.
  Becher, Neubert, Pecjak, JHEP0701, 076 (2007) Becher, Neubert, PRL97, 082001 (2006)
- Pointed out awhile ago that factorization and renormalization group invariance leads to exponentiation of Sudakov logs
  Contopanagos, Laenen, Sterman, NPB484, 303 (1997); Forte, Ridolfi, NPB650, 229 (2003)
- Process has been applied to
  - Drell-Yan Becher, Neubert, Xu, JHEP0807, 030 (2008)
  - Higgs production Ahrens, Becher, Neubert, Yang, PRD79, 033013(2009); EPJ C62, 333 (2009); PLB698, 271 (2011)
  - Direct photon production Becher, Schwartz, JHEP1002, 040 (2010)
  - Slepton production Broggio, Neubert, Vernazza, JHEP1205, 151 (2012)
  - $t\bar{t}$ production Ahrens, Ferroglia, Neubert, Pecjak, Yang, JHEP1009, 097 (2010); JHEP1109, 070 (2011), PLB703, 125 (2011)

Hard Piece for W^+W^-

- Hard function calculated via Wilson coefficient of SCET operators, C_{WW}:

$$\mathcal{H}(M_{WW}, \cos\theta, \mu_h) = |C_{WW}(M_{WW}, \cos\theta, \mu_h)\mathcal{O}_{WW}|^2$$

- C_{WW} calculated by matching SCET onto full QCD at a hard scale $\mu_h \sim M_{WW}$:

$$\mathcal{M}^{\text{ren}}_{\text{virt}}(\varepsilon, M_{WW}, \cos\theta, \mu_h) = Z(\varepsilon, M_{WW}, \mu_h)C_{WW}(M_{WW}, \cos\theta, \mu_h)\mathcal{O}_{WW}$$

  - $\mathcal{M}^{\text{ren}}_{\text{virt}}$ is the renormalized QCD amplitude.
  - Z is the SCET renormalization constant.

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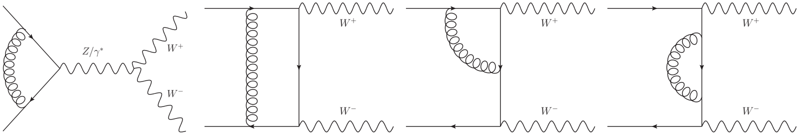
  - $\mathcal{M}^{\text{ren}}_{\text{virt}}$ is the renormalized QCD amplitude.
  - Z is the SCET renormalization constant.
- In SCET and dimensional regularization all loops are scaleless and vanish.
- IR and UV divergences in SCET cancel.
- Hence, IR pole structure of $\mathcal{M}^{\text{ren}}$ match the pole structure of Z.
- As expected since in IR regime QCD and SCET describe the same physics.
  - IR poles of QCD = IR poles of SCET = UV poles of SCET

Hard Piece



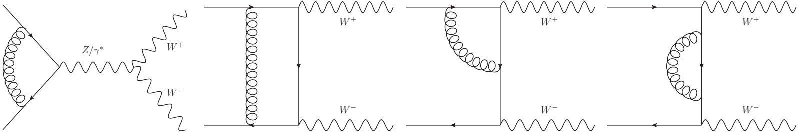
- For W^+W^-, at one loop, have the amplitude squared Frixione, NPB410, 280 (1993)

$$|\mathcal{M}^{\rm ren}_{\rm virt}|^2 = |\mathcal{M}^{Born}|^2 - \frac{\alpha_s C_F}{4\pi}\left(\frac{4\pi\mu_h^2}{M_{WW}^2}\right)^{\varepsilon}\Gamma(1+\varepsilon)\left(\frac{4}{\varepsilon^2}+\frac{6}{\varepsilon}\right)|\mathcal{M}^{Born}|^2 + |\mathcal{M}^{v,reg}|^2$$

- Since dealing with $q\bar{q}$ initial state, SCET renormalization constant same as Drell-Yan:

$$Z(M_{WW},\cos\theta,\mu_h) = 1 - \frac{\alpha_s C_F}{4\pi}(4\pi)^{\varepsilon}e^{-\varepsilon\gamma_E}\left(\frac{2}{\varepsilon^2}+\frac{2}{\varepsilon}\ln\frac{\mu_h^2}{-M_{WW}^2}+\frac{3}{\varepsilon}\right)$$

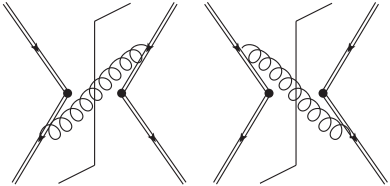
Hard Piece



- After renormalizing the SCET operators:

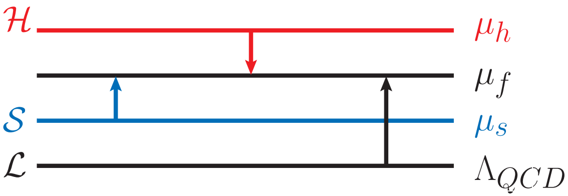
$$\mathcal{H}(M_{WW},\cos\theta,\mu_h) = |Z^{-1}(\varepsilon,M_{WW},\mu_h)\mathcal{M}^{\mathrm{ren}}|^2$$
$$= \frac{\beta}{8\pi M_{WW}}\left\{\left[1-\frac{\alpha_s C_F}{2\pi}\left(\ln^2\frac{\mu_h^2}{M_{WW}^2}+3\ln\frac{\mu_h^2}{M_{WW}^2}+\frac{\pi^2}{6}\right)\right]|\mathcal{M}^{Born}|^2 + |\mathcal{M}^{v,reg}|^2\right\}$$

Soft Piece



- Soft function describes soft gluon emission.
- After “decoupling transformation" in SCET soft and collinear fields do not interact.
- Double lines are soft Wilson lines, and soft function ends up being a Wilson loop.
- Introduce the soft scale μ_s where assume we can do this calculation perturbatively.
- The soft function can calculated via one loop Feynman diagrams using eikonal approximation and cut gluon propagators.
- Final piece is RGEs.

RG Running



- Know hard function RGE:

$$\frac{d}{d\ln\mu}\mathcal{H}(M_{WW},\cos\theta,\mu) = 2\left[\Gamma_{\rm Cusp}(\alpha_s)\ln\frac{M_{WW}^2}{\mu^2} + \gamma^V(\alpha_s)\right]\mathcal{H}(M_{WW},\cos\theta,\mu)$$

- In limit $x\to 1$, have PDF evolution:

$$\begin{aligned}\frac{d}{d\ln\mu}f_{q/N}(x,\mu) &= \int_z^1 P_{q\leftarrow q}(z)f_{q/N}(x/z,\mu)\\ P_{q\leftarrow q}(z) &= \frac{2\Gamma_{Cusp}(\alpha_s)}{[1-z]_+} + 2\gamma^\phi(\alpha_s)\delta(1-z) + \ldots.\end{aligned}$$

- Total cross section scale-invariant $\Rightarrow$ solve for soft function running in terms of PDFs and hard function.
- Evaluate each piece at appropriate scale then RG evolve to common scale.

Resummed Cross Section

- Since renormalization of SCET operator the same as Drell-Yan, can use previous results to finish calculation Becher, Neubert, Xu, JHEP 0807, 030 (2008):

$$\frac{d\sigma^{Thresh}}{dM_{WW}\,d\cos\theta} = \int_\tau^1 \frac{dz}{z} C(z, M_{WW}, \cos\theta, \mu_f)\mathcal{L}\left(\frac{\tau}{z}, \mu_f\right)$$

$$C(z, M_{WW}, \cos\theta\mu_f) = \mathcal{H}(M_{WW}, \mu_h) U(M_{WW}, \mu_h, \mu_s, \mu_f)\frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$

$$\times \quad \tilde{s}\left(\ln\frac{M_{WW}^2(1-z)^2}{\mu_s^2 z} + \partial_\eta, \mu_s\right)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

- U arises from RGE running and contains exponentiated logs:

$$\ln U(M_{WW}, \mu_h, \mu_s, \mu_f) = 4S(\mu_h, \mu_s) - 2a_{\gamma^V}(\mu_h, \mu_s) + 4a_{\gamma^\phi}(\mu_s, \mu_f) - 2a_\Gamma(\mu_h, \mu_s)\ln\frac{M_{WW}^2}{\mu_h{}^2}$$

- S is the Sudakov exponent, and a are subleading exponents.
- $\eta = 2a_\Gamma(\mu_s, \mu_f)$

Precision of Resummed Results

Order	Accuracy: $\alpha_s^n \ln^m(\mu_s/M_{WW})$	Γ_{cusp}	γ^h, γ^ϕ	$H, \tilde{s}$
NLL	$2n-1 \leq m \leq 2n$	2-loop	1-loop	tree
NNLL	$2n-3 \leq m \leq 2n$	3-loop	2-loop	1-loop

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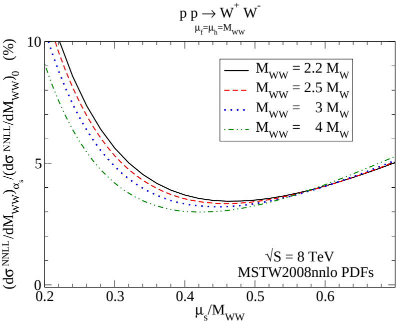
- S is the Sudakov exponent, and a are subleading exponents:

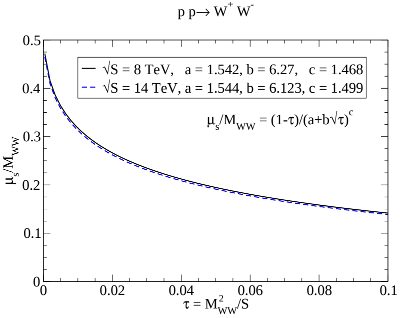
$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\rm Cusp}}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \qquad a_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\rm Cusp}(\alpha_s)}{\beta(\alpha)}$$

Scale Choice



- Soft scale:
  - The soft scale is chosen by minimizing the one-loop contribution Becher, Neubert, Xu, JHEP 0807, 030 (2008).
  - Enforcing $\mu_s \propto (1-\tau)$ as $\tau \to 1$: $\frac{\mu_s^{\min}}{M_{WW}} = \frac{1-\tau}{(1.542+6.270\sqrt{\tau})^{1.468}}$
- Hard scale:
  - Hard scale set to scale of hard scattering process: $\mu_h = M_{WW}$.

Scale Choice



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- Hard scale:
  - Hard scale set to scale of hard scattering process: $\mu_h = M_{WW}$.

Matching

- Resummed result is only valid at threshold.
- Fixed order calculation valid away from threshold.
- Need to combine these two results to obtain result valid for all z:

$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{Leading}$$

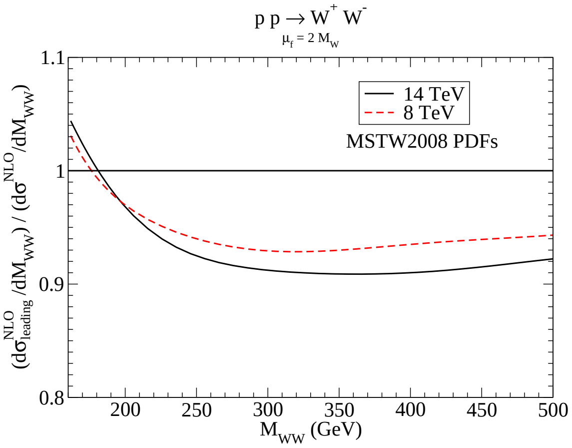
- Have introduced the leading singularity term:

$$d\sigma^{Leading} = d\sigma^{Thresh}|_{\mu_s=\mu_h=\mu_f}$$

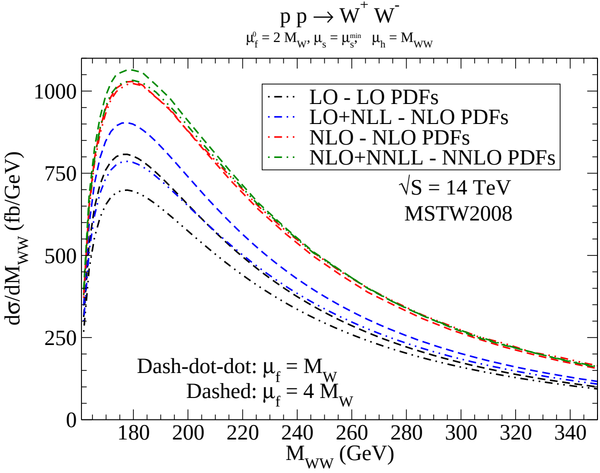
- The leading singularity is subtracted to prevent double counting between the fixed order and resummed results.

Leading Logs

- Hopefully resumming large logs capture most of higher order correction.
- Ratio of leading singularities to total NLO result.
- Here $d\sigma^{Leading}$ includes the NLO coefficient function.

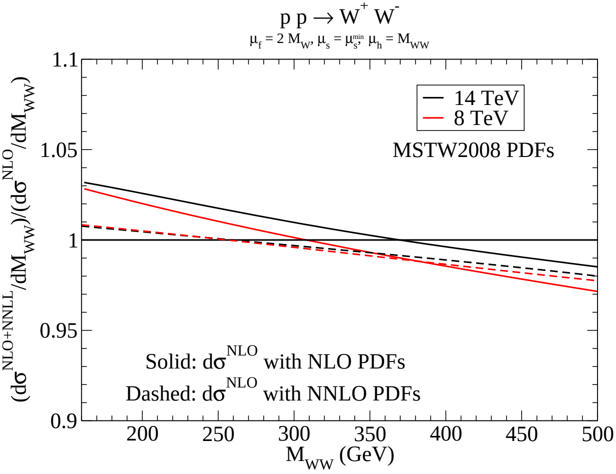


Invariant Mass Distribution



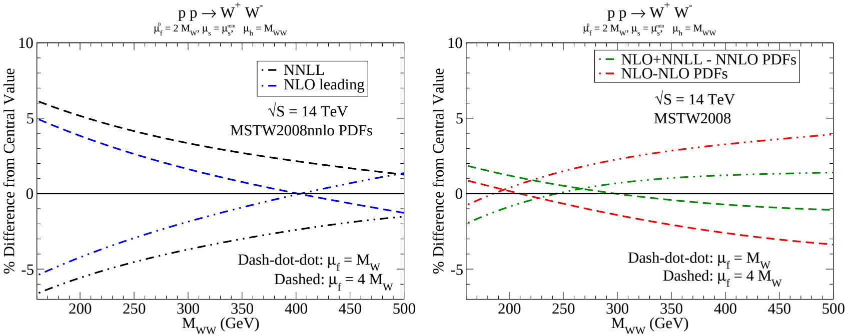
- Increase in cross section at peak.

Invariant Mass Distribution



- Much of the change in shape from NNLO pdfs.

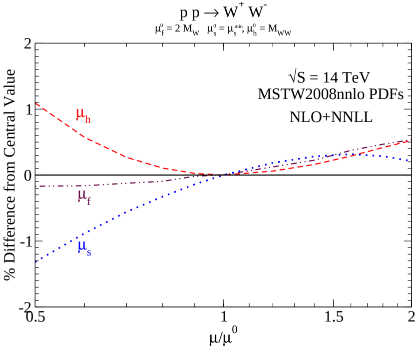
Factorization Scale Dependence



$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{Leading}$$

- For $M_{WW} \lesssim 400$ GeV, cancellation between NNLL resummed and leading singularity.
- For $M_{WW} \gtrsim 190$ GeV, cancellation between NNLL resummed and NLO contributions.

Full Scale Dependence



$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{Leading}$$

- In matched contribution, factorization scale dependence cancels among the three pieces.
- Factorization scale dependence greater than hard and soft scale dependencies.
- NLO scale dependence: $117.9^{+1.4\%}_{-0.9\%}$ pb
- Factorization scale dependence decreases significantly.

Approximate NNLO

- From knowledge of hard and soft functions, can construct an approximate NNLO result.
- Expand the scattering kernel in a power series:

$$\begin{aligned} C(z, M_{WW}, \cos\theta, \mu_f) &= C_0(z, M_{WW}, \cos\theta, \mu_f) + \frac{\alpha_s}{4\pi} C_1(z, M_{WW}, \cos\theta, \mu_f) \\ &+ \left(\frac{\alpha_s}{4\pi}\right)^2 C_2(z, M_{WW}, \cos\theta, \mu_f) \end{aligned}$$

- Scattering kernel known fully up to NLO.
- NNLO piece can be approximated by expanding the scattering kernel in the threshold limit at the factorization scale $\mu_h = \mu_s = \mu_f$:

$$C(z, M_{WW}, \cos\theta, \mu_f) = \mathcal{H}(M_{WW}, \cos\theta, \mu_f)\mathcal{S}(\sqrt{\hat{s}}(1-z), \mu_f)$$

- Take power expansions of hard and soft functions, then

$$C_2 = \mathcal{H}_0 \mathcal{S}_2 + \mathcal{H}_1 \mathcal{S}_1 + \mathcal{H}_2 \mathcal{S}_0$$

Approximate NNLO

- Soft function evaluated at $\mu_s = \mu_f$

$$\mathcal{S}(\sqrt{\hat{s}}(1-z),\mu_f) = \exp[-4S(\mu_s,\mu_f)+2a_{\gamma^s}(\mu_s,\mu_f)]\tilde{s}(\partial_\eta,\mu_s)\frac{e^{-2\gamma_e\eta}}{\Gamma(2\eta)}\left(\frac{\hat{s}}{\mu_s{}^2}\right)^{\eta}\frac{1}{(1-z)^{1-2\eta}}$$
$$\to \tilde{s}(\partial_\eta,\mu_f)\frac{e^{-2\gamma_e\eta}}{\Gamma(2\eta)}\left(\frac{\hat{s}}{\mu_f{}^2}\right)^{\eta}\frac{1}{(1-z)^{1-2\eta}}\Bigg|_{\eta=0}$$

  - Soft function same as Drell-Yan, known to NNLO Becher, Neubert, Xu, JHEP 0807, 030 (2008).
  - Captures behavior that is singular as $z \to 1$, that $\delta(1-z)$ and "plus-functions":

$$\left[\frac{\ln^n(1-z)}{1-z}\right]_+$$

Approximate NNLO

- Hard function:
  - Known fully to NLO.
  - Can approximate NNLO piece from RG running.
  - Expand NNLO piece of hard function:

$$\mathcal{H}_2 = \sum_{n=0}^{4} h^{(2,n)} L_{WW}^n, \quad L_{WW} = \ln \frac{M_{WW}^2}{\mu_f^2}$$

  - Insert into RGE and can solve for NNLO scale dependent pieces in terms of LO and NLO pieces:

$$\frac{d}{d\ln\mu}\mathcal{H}(M_{WW}, \cos\theta, \mu) = 2\left[\Gamma_{\text{Cusp}}(\alpha_s)\ln\frac{M_{WW}^2}{\mu^2} + \gamma^V(\alpha_s)\right]\mathcal{H}(M_{WW}, \cos\theta, \mu)$$

  - $h^{(2,0)}$ only determined via full calculation.

Approximate NNLO

- Have approximate NNLO hard piece:

$$\mathcal{H}_2^{\text{approx}} = \sum_{n=1}^{4} h^{(2,n)} L_{WW}^n$$

  - Independent of scale up to $\mathcal{O}(\alpha_s^3)$
  - Pieces missing at $\mathcal{O}(\alpha_s^2) \Rightarrow$ simple scale variation underestimates uncertainty.
  - Introduce new scale $Q_h \sim M_{WW}$ and consider replacement

$$L_{WW} = \ln \frac{M_{WW}^2}{\mu_f^2} \rightarrow \ln \frac{Q_h^2}{\mu_f^2}$$

  - $\mathcal{O}(1)$ variations in Q_h corresponds to variations in scale independent piece.
  - Use log expansion and vary Q_h to estimate $\mathcal{O}(\alpha_s^2)$ variation.

Approximate NNLO

- Reproduce all singular pieces up to a scaled independent piece proportional to $\delta(1-z)$:

$$
\begin{aligned}
C_2^{approx} &= \mathcal{H}_0\mathcal{S}_2 + \mathcal{H}_1\mathcal{S}_1 + \mathcal{H}_2^{approx}\,\mathcal{S}_0 \\
&= \sum_{n=0}^{3} D^{(3)}\left[\frac{\ln^n(1-z)}{1-z}\right]_+ + R^{(0)}_{approx}\,\delta(1-z)
\end{aligned}
$$

Total Cross Section

σ(pb)	$\sqrt{S} = 7$ TeV	$\sqrt{S} = 8$ TeV	$\sqrt{S} = 13$ TeV	$\sqrt{S}$=14 TeV
σ^{NLO}	$45.7^{+1.5}_{-1.1}$	$55.7^{+1.7}_{-1.2}$	$110.6^{+2.5}_{-1.6}$	$122.2^{+2.5}_{-1.8}$
σ^{gg}	$1.0^{+0.3}_{-0.2}$	$1.3^{+0.4}_{-0.3}$	$3.5^{+0.9}_{-0.7}$	$4.1^{+0.9}_{-0.7}$
$\sigma'^{NLO+NNLL}$	$45.9^{+0.5}_{-0.6}$	$56.1^{+0.7}_{-0.8}$	$111.7^{+1.8}_{-1.6}$	$123.6^{+2.0}_{-1.8}$
σ'^{NNLO}_{approx}	$46.0^{+0.4}_{-0.047}$	$56.2^{+0.6}_{-0.1}$	$111.8^{+1.7}_{-1.1}$	$123.7^{+1.8}_{-1.2}$

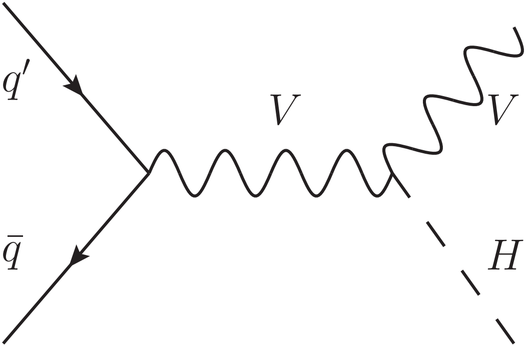
$$\mu_f^0 = 2M_W\,, \quad \mu_h^0 = M_{WW}\,, \quad \mu_s^0 = \mu_s^{\rm min}$$

σ(pb)	$\sqrt{S} = 7$ TeV	$\sqrt{S} = 8$ TeV	$\sqrt{S} = 13$ TeV	$\sqrt{S}$=14 TeV
σ^{NLO}	$44.8^{+1.2}_{-0.9}$	$54.7^{+1.4}_{-1.0}$	$108.8^{+1.2}_{-1.3}$	$120.3^{+2.0}_{-1.3}$
σ^{gg}	$0.9^{+0.2}_{-0.2}$	$1.2^{+0.3}_{-0.1}$	$3.3^{+0.8}_{-0.6}$	$3.7^{+0.7}_{-0.6}$
$\sigma'^{NLO+NNLL}$	$45.6^{+0.6}_{-0.6}$	$55.8^{+0.7}_{-0.8}$	$111.4^{+2.0}_{-1.8}$	$123.1^{+2.1}_{-2.0}$
σ'^{NNLO}_{approx}	$45.7^{+0.4}_{-0.04}$	$55.9^{+0.5}_{-0}$	$111.5^{+1.6}_{-1.0}$	$123.3^{+1.7}_{-1.2}$

$$\mu_f^0 = M_{WW}\,, \quad \mu_h^0 = M_{WW}\,, \quad \mu_s^0 = \mu_s^{\rm min}$$

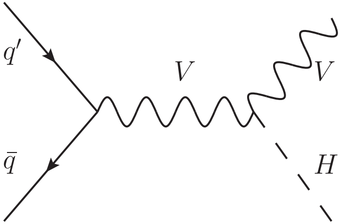
- Approximate NNLO and matched results agree well.
- Increases NLO cross section by $1-3\%$

Higgs Associated Production



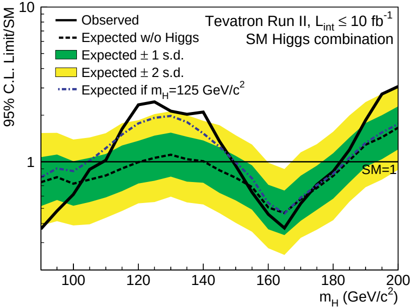
Sally Dawson, Tao Han, Wai-Kin Lai, Adam Leibovich, PRD86 (2012) 074007

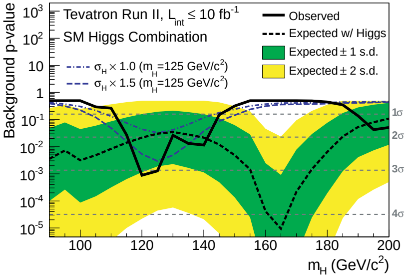
Motivation to Study *VH*



- Need precise predictions.
- Previous section dealt with important background, now concentrate on a Higgs signal.
- Production in association with a vector boson also important.
  - Directly probes *VVH* coupling.
  - Tevatron excesses observed in this channel.
  - For LHC, important for observing $H \to b\bar{b}$ channel. Butterworth et al, PRL 100, 242001 (2008)

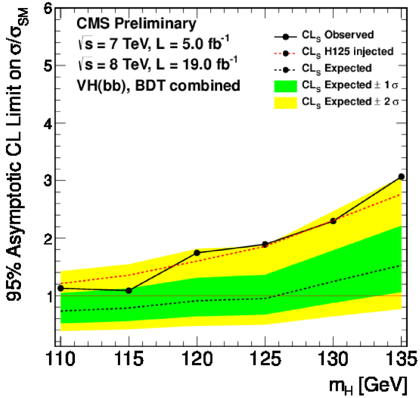
Tevatron Results

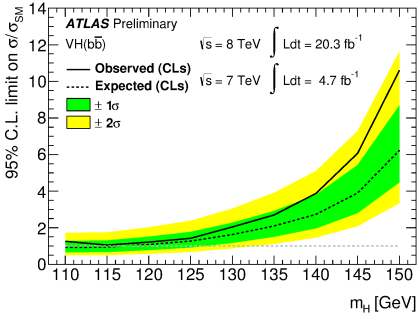




- Tevatron Higgs signal from *VH* production.
- 3σ excess for $M_h \sim 120 - 125$ GeV

LHC Results





- CMS has $\sim 2\sigma$ signal.

Current Status

- Known to NNLO in QCD. Brein, Djouadi, Harlander, PLB579 149 (2004); Brein et al, EPJ C72 1868 (2012)
  - NLO increase cross sections by $\sim 20\%$ Han, Willenbrock, PLB2737, 167 (1991)
    Baer, Bailey, Owens, PRD47, 2730 (1993)
    Ohnemus, Stirling, PRD47, 2722 (1993)
  - NNLO increases WH by another $1-2\%$
  - NNLO increases ZH by another $7-8\%$
  - Difference accounted for by $gg \to ZH$ box diagram contributing $\sim 5\%$ to ZH
- NLO electroweak corrections also known Ciccolini, Dittmaier, Kramer, PRD68, 073003 (2003)
  - Decrease cross section by $5-10\%$

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- NLO electroweak corrections also known Ciccolini, Dittmaier, Kramer, PRD68, 073003 (2003)
  - Decrease cross section by $5-10\%$
- As with W^+W^- and Drell-Yan, has $q\bar{q}$ initial state.
- For threshold resummation, soft function same as in Drell-Yan.
- Recalculate hard function.
- Take into consideration different phase space.

Scale Choice

- Choose soft scale to minimize effects of higher order corrections
  - $\mu_s^I = \dfrac{M_{VH}(1-\tau)}{2\sqrt{1+100\tau}}$ chosen to minimize 1-loop correction to soft piece.
  - $\mu_s^{II} = \dfrac{M_{VH}(1-\tau)}{0.9+12\tau}$ chosen when 1-loop correction drops below 10%
- Hard scale: $\mu_h = 2M_{VH}$

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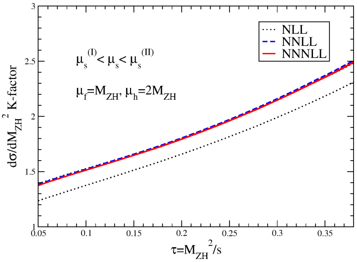
  - $\mu_s^{II} = \dfrac{M_{VH}(1-\tau)}{0.9+12\tau}$ chosen when 1-loop correction drops below 10%
- Hard scale: $\mu_h = 2M_{VH}$
- Analyze scale variation via K-factor:

$$\frac{d\sigma}{dM_{VH}^2} \equiv K \frac{d\sigma}{dM_{VH}^2}\Big|_{\mathrm{LO}}$$

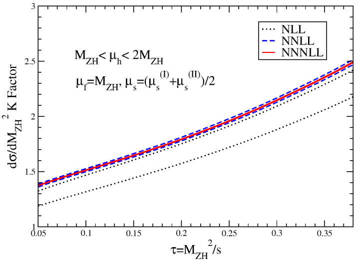
  - $d\sigma/dM_{VH}^2$ is a higher order QCD distribution

Scale dependence

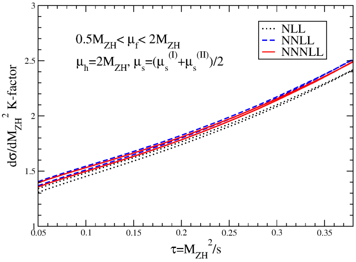
Soft Scale:



Hard Scale:

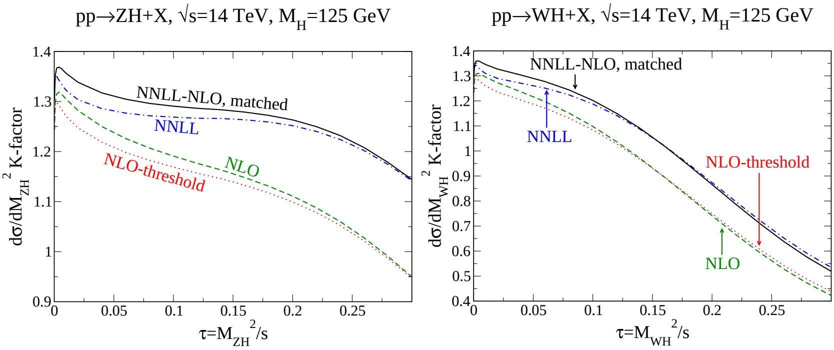


Factorization Scale:



- Scale variation of resummed piece.
- Black dotted: NLL
  Blue: NNLL
  Red: NNNLL
- Area between curves indicates scale variation.
- All cross sections evaluated using MSTW2008NNLO pdfs

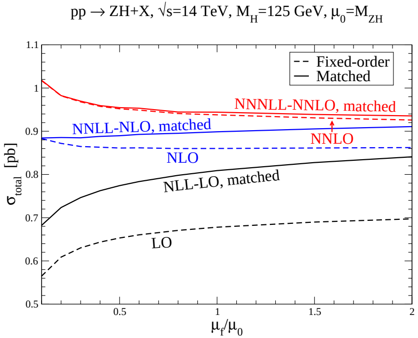
Invariant Mass Distribution for *VH*

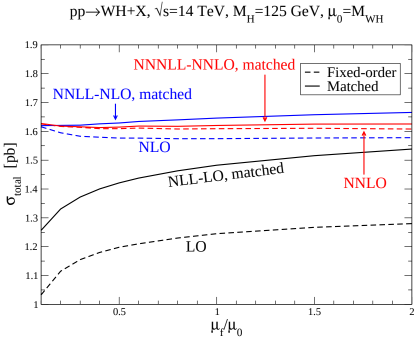


$$d\sigma^{matched} = d\sigma^{Thresh} + d\sigma^{F.O.} - d\sigma^{NLO-threshold}$$

- K-factor evaluated with LO pdfs for LO distribution and NLO pdfs for all others.

14 TeV Cross Sections





- NNNLL has little effect.
- NNLL increases cross section $\sim 7\%$ for *ZH* and $\sim 3\%$ for *WH*
- Including threshold logs does not introduce added uncertainty.

- $\mu_s = \frac{1}{2}(\mu_s^I + \mu_s^{II})$
- $\mu_h = 2M_{VH}$
- MSTW2008 68% CL
- Use VH@NNLO for fixed order NNLO result
  Brein, Djouadi, Harlander, PLB579, 149 (2004)

Transverse Momentum Resummation

- When *VH* system has low transverse momentum, $p_T \ll M_{VH}$, large logs also appear:

$$\alpha_s^n \ln^{2n-1}\left(\frac{M_{VH}^2}{p_T^2}\right)$$

- Apply impact parameter resummation to partonic cross section:
CSS, Nucl.Phys. B250, 199 (1985); Bozzi *et al*, Nucl.Phys. B737, 73 (2006)

$$M_{VH}^2 \frac{d\hat{\sigma}_{VH}^{resum}}{dM_{VH}^2 dp_{T,VH}^2} = \frac{M_{VH}^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bp_{T,VH}) W^{VH}(b, M_{VH}, \hat{s}, \mu_r, \mu_f)$$

- In small p_T limit, also have a factorization of hard and soft pieces:

$$\begin{aligned} W_N^{VH}(b, M_{VH}, \mu_r, \mu_f) \quad = \quad & H_N^{VH}\left(M_{VH}, \alpha_s(\mu_r), \frac{M_{VH}}{\mu_r}, \frac{M_{VH}}{\mu_f}, \frac{M_{VH}}{Q}\right) \\ & \times \exp\left\{G_N\left(\alpha_s(\mu_r), L, \frac{M_{VH}}{\mu_r}, \frac{M_{VH}}{Q}\right)\right\} \end{aligned}$$

  - Small p_T corresponds to large impact parameter b
  - $L = \ln(Q^2 b^2 / b_0^2)$, have exponentiated large logs.
  - $Q \sim M_{VH}$ is the “renormalization scale"

Transverse Momentum Resummation

$$W_N^{VH}(b, M_{VH}, \mu_r, \mu_f) = H_N^{VH}\left(M_{VH}, \alpha_s(\mu_r), \frac{M_{VH}}{\mu_r}, \frac{M_{VH}}{\mu_f}, \frac{M_{VH}}{Q}\right) \times \exp\left\{G_N\left(\alpha_s(\mu_r), L, \frac{M_{VH}}{\mu_r}, \frac{M_{VH}}{Q}\right)\right\}$$

- H_N^{VH} has a power expansion in α_s:

$$H_N^{VH} = \sigma_0(\alpha_s, M_{VH})\left\{1 + \frac{\alpha_s}{\pi} H_N^{VH(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 H_N^{VH(2)} + \cdots\right\}$$

- G_N has a log expansion:

$$G_N = L g_N^1(\alpha_s L) + g_N^2(\alpha_s L) + \left(\frac{\alpha_s}{\pi}\right) g_N^3(\alpha_s L) + \cdots$$

  - LL ($\alpha_s^n L^{n+1}$) terms: $L g_N^1$
  - NLL ($\alpha_s^n L^N$) terms: g_N^2, etc.

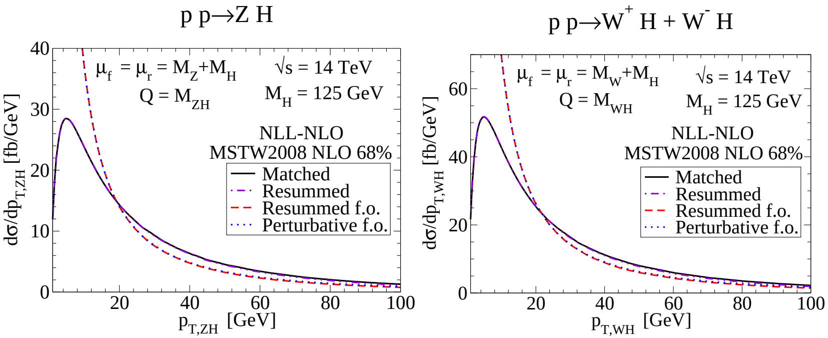
Matching

- Transverse momentum resummed result only valid for $p_T \ll M_{VH}$
- Fixed order results valid for $p_T \sim M_{VH}$ and no large logs.
- Need to match the two results:

$$\frac{d\hat{\sigma}_{VH}}{dM^2_{VH}dp^2_{T,VH}} = \frac{d\hat{\sigma}^{resum}_{VH}}{dM^2_{VH}dp^2_{T,VH}} + \left[\frac{d\hat{\sigma}_{VH}}{dM^2_{VH}dp^2_{T,VH}}\right]_{f.o} - \left[\frac{d\hat{\sigma}^{resum}_{VH}}{dM^2_{VH}dp^2_{T,VH}}\right]_{f.o}$$

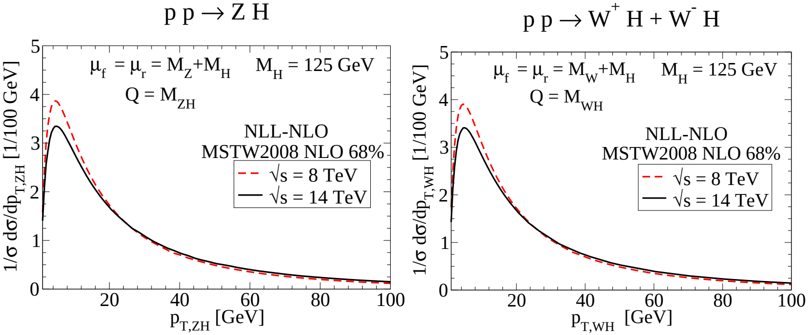
    - Subscript $f.o.$ indicates fixed order expansion.
- Subtraction of $[d\hat{\sigma}^{resum}]_{f.o.}$ cancels large logs in fixed order cross section at low p_T.
- Reproduce correct cross section at a given order.

Transverse Momentum Distribution



- As expected, perturbative expansions blows up at $p_T \to 0$

Normalized Transverse Momentum Distribution



- Longer tail at 14 TeV, more available energy for harder emission.

Conclusions

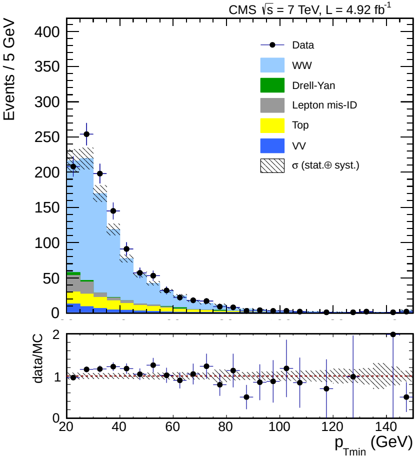
- Have discovered a Higgs boson.
- Now that we have it, important to measure its properties as well as possible.
- Need solid theoretical and experimental understanding of signal as well as backgrounds.
- Also have an anomaly in W^+W^- production at 7 and 8 TeV.
- Important to check reliability of perturbative calculations.
- Large threshold logs are expected to be dominant contributions higher order contributions.

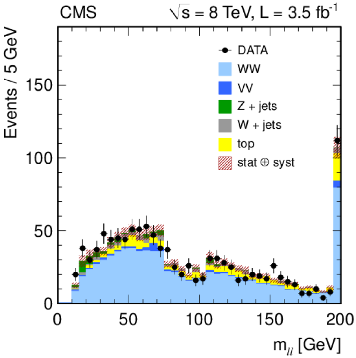
Conclusions

- Have discovered a Higgs boson.
- Now that we have it, important to measure its properties as well as possible.
- Need solid theoretical and experimental understanding of signal as well as backgrounds.
- Also have an anomaly in W^+W^- production at 7 and 8 TeV.
- Important to check reliability of perturbative calculations.
- Large threshold logs are expected to be dominant contributions higher order contributions.
- Calculated threshold resummed and approximate NNLO cross sections for SM W^+W^- production.
  - Threshold resummation calculated to NNLL
  - Both increase NLO cross section by $1-2\%$.
  - Not enough to explain apparent anomaly in W^+W^- cross section.
  - Invariant mass distribution slightly increased near peak.
- Calculated threshold resummed cross section for VH production.
  - NNLL resummation increase NLO ZH cross section $\sim 7\%$ and WH by $\sim 3\%$.
  - NNNLL resummation makes little difference to NNLO VH production.
- Fixed order perturbation theory under control.

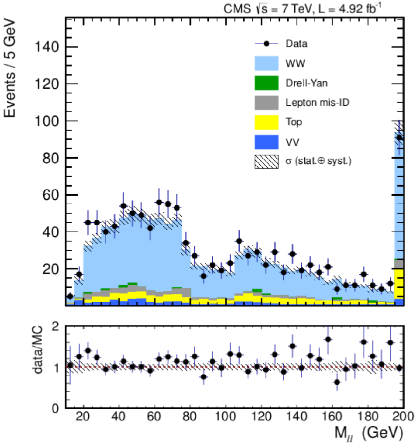
BACKUP SLIDES

More CMS


CMS √s = 7 TeV, L = 4.92 fb-1
Events / 5 GeV
Data
WW
Drell-Yan
Lepton mis-ID
Top
VV
σ (stat.⊕ syst.)
data/MC
p_Tmin (GeV)

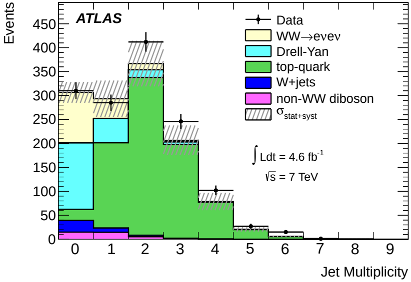

CMS
√s = 8 TeV, L = 3.5 fb-1
Events / 5 GeV
DATA
WW
VV
Z + jets
W + jets
top
stat ⊕ syst
m_ll [GeV]

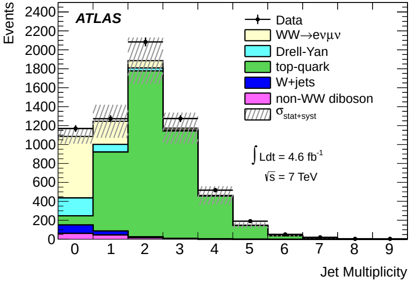
More CMS

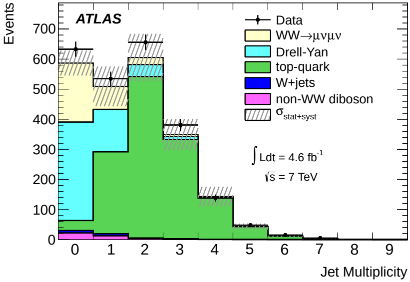


CMS √s = 7 TeV, L = 4.92 fb⁻¹
Events / 5 GeV
Data
WW
Drell-Yan
Lepton mis-ID
Top
VV
σ (stat.⊕ syst.)
data/MC
M_ll (GeV)

ATLAS Jet Multiplicity for W^+W^-







Jet Cuts

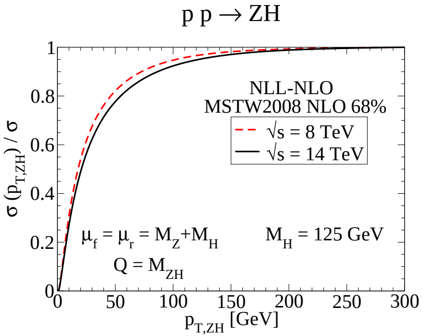
- Jet vetoes can be important for eliminating background.
- Vetoing jets with a minimum p_T may be approximated by placing an upper limit on $p_{T,VH}$
- As shown, the perturbative calculation breaks down in this regime and the soft-gluon resummation is needed.
- There has been much recent work on the systematic resummation of the large logs associated with jet vetoes.

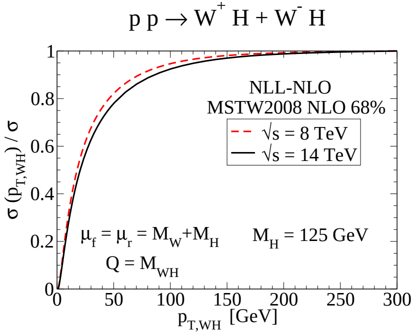
Berger *et al*, JHEP1104, 092 (2011)
Becher, Neubert, JHEP1207, 108 (2012)
Liu, Petriello, PRD87, 014018 (2013); PRD87, 094027 (2013)
Banfi, Salam, Zanderighi, JHEP1206, 159 (2012)
Tackmann, Walsh, Zuberi, PRD86, 053011 (2012)
Stewart, Tackmann, Walsh, Zuberi, 1307.1808

- “Poor man’s" jet veto:

$$\sigma(p_{T,VH}) = \int_0^{p_{T,VH}} dq_{T,VH} \frac{d\sigma}{dq_{T,VH}}$$

Transverse Momentum Cut



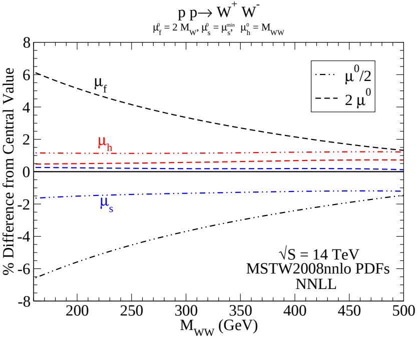


$1-\dfrac{\sigma(p_{T,VH})}{\sigma}$	8 TeV	14 TeV
$p_{T,VH} < 20$ GeV	$\sim 45\%$	$\sim 50\%$
$p_{T,VH} < 30$ GeV	$\sim 33\%$	$\sim 37\%$

Conclusions

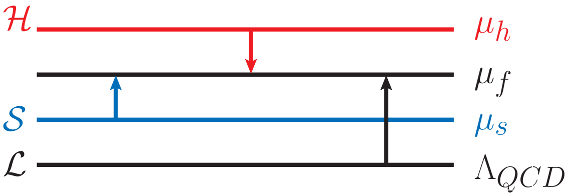
- Performed the transverse momentum resummation of the VH system.
- Calculated the effects on the NLO cross sections of placing a cut on the p_T of the VH system. Expect such a cut to approximate a jet veto.
- Found p_T cut decreased NLO cross section by $33\% - 50\%$.
- Expect such a cut to approximate a jet veto.

Full Scale Dependence for W^+W^-



- Factorization scale dependence greater than hard and soft scale dependencies.

RG Running



- Hard function solution Becher, Neubert, Xu, JHEP 0807, 030 (2008):

$$\frac{\mathcal{H}(M_{WW},\cos\theta,\mu_f)}{\mathcal{H}(M_{WW},\cos\theta,\mu_h)} = \exp[4S(\mu_h,\mu_f) - 2a_{\gamma^V}(\mu_h,\mu_f)]\left(\frac{M_{WW}^2}{{\mu_h}^2}\right)^{-a_\Gamma(\mu_h,\mu_f)}$$

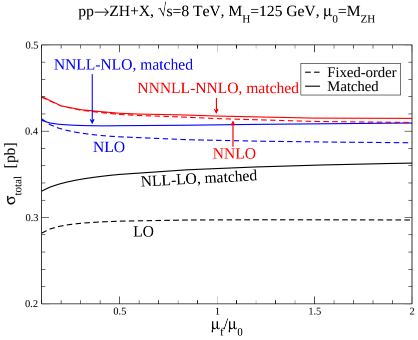
- Soft function solution with $\gamma^S = 2\gamma^\phi + \gamma^V$:

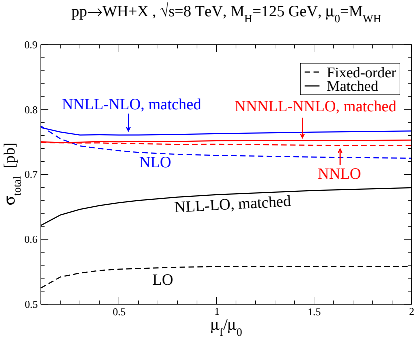
$$\mathcal{S}(\sqrt{\hat{s}}(1-z),\mu_f) = \exp[-4S(\mu_s,\mu_f) + 2a_{\gamma^S}(\mu_s,\mu_f)]\tilde{s}(\partial_\eta,\mu_s)\frac{e^{-2\gamma_e\eta}}{\Gamma(2\eta)}\left(\frac{\hat{s}}{{\mu_s}^2}\right)^\eta \frac{1}{(1-z)^{1-2\eta}}$$

- S is the Sudakov exponent, and a are subleading exponents:

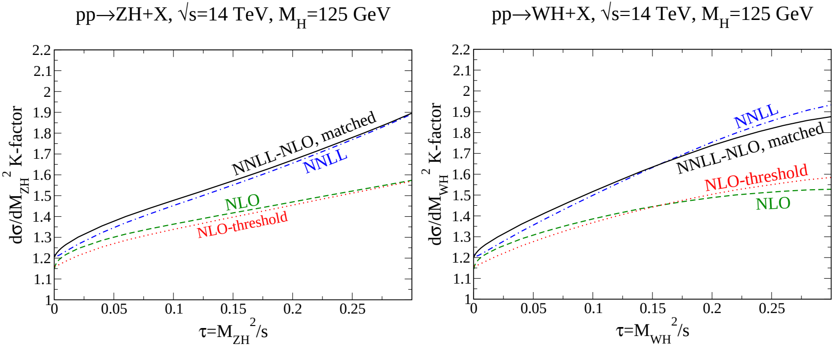
$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\text{Cusp}}}{\beta(\alpha)}\int_{\alpha_s(\nu)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} \qquad \eta = 2a_\Gamma(\mu_s,\mu_f) = -2\int_{\alpha_s(\mu_s)}^{\alpha_s(\mu_f)} d\alpha \frac{\Gamma_{\text{Cusp}}(\alpha_s)}{\beta(\alpha)}$$

8 TeV Cross Sections For *VH*





Invariant Mass Distribution for *VH*



- K-factor evaluated using NLO pdfs for all distributions.

SCET matching

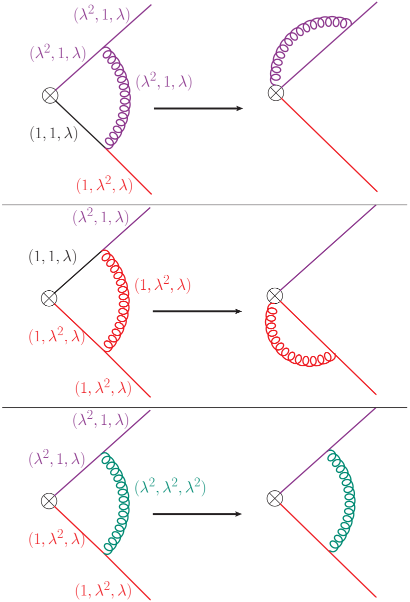
- SCET is an effective theory made of field that are described by soft and collinear fields.
- Basis:
  - Reference vectors: $n^\mu = (1,0,0,1), \quad \bar{n}^\mu = (1,0,0,-1)$
  - Decompose four vectors: $p^\mu = p_+ \frac{\bar{n}^\mu}{2} + p_- \frac{n^\mu}{2} + p_\perp^2 = (p_+, p_-, p_\perp)$
  - Where $p_+ = p \cdot n, \quad p_- = p \cdot \bar{n}$
- Momentum classification:

$$
\begin{array}{ll}
\text{Collinear in } n \text{ direction:} & p_n \sim Q(\lambda^2, 1, \lambda) \Rightarrow p_n^2 \sim Q^2\lambda^2 \\
\text{Collinear in } \bar{n} \text{ direction:} & p_{\bar{n}} \sim Q(1, \lambda^2, \lambda) \Rightarrow p_{\bar{n}}^2 \sim Q^2\lambda^2 \\
\text{Soft:} & p_s \sim Q(\lambda^2, \lambda^2, \lambda^2) \Rightarrow p_s^2 \sim Q^2\lambda^4 \\
\text{Hard:} & p_h \sim Q(1,1,1) \Rightarrow p_h^2 \sim Q^2
\end{array}
$$

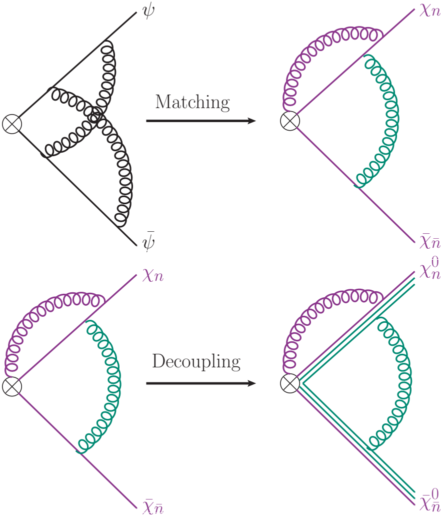
SCET matching

- Introduce fields:
  - Dirac field with momentum in direction n: $\psi \to \xi_n + \eta$
  - $\xi_n = \frac{\not{n}\bar{\not{n}}}{4}\psi \qquad \eta = \frac{\bar{\not{n}}\not{n}}{4}\psi$
  - Similarly $D = \partial + igA_c + igA_s$
  - ξ_n is the collinear fermion in the n direction.
  - From propagators find scaling $\xi \sim \lambda$, $\eta \sim \lambda^2$
  - Similarly, can find gluon field components scale like their momentum
- Also useful to introduce Wilson lines:
  - Collinear Wilson line: $W_n(x) = P\exp[ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_c(x+s\bar{n})]$
  - Then combination of $W_n(x)\xi_n$ is gauge invariant under SCET.
  - Soft Wilson line: $S_n(x) = P\exp[ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_s(x+s\bar{n})]$
  - Soft Wilson line useful for “decoupling transformation" that decouples soft and collinear fields.

SCET Matching



SCET Matching



$$\bar\psi\Gamma\psi \to \bar\xi_{\bar n} W_{\bar n} r \Gamma W^\dagger_{\bar n} \xi_n \equiv \bar\chi_{\bar n}\Gamma\chi_n$$

$$\bar\chi_{\bar n}\Gamma\chi_n = \bar\chi^0_{\bar n} S^\dagger_{\bar n}\Gamma S_n \chi^0_n$$

After decoupling tranformation, collinear and soft fields do not interact.